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Theoretical 3: Tornado

CONSTANTS AND DATA:		
Gravitational acceleration	:	$g = 9.8$ m/s^2
Air density	:	$\rho_{\text{Air}} = 1.2$ kg/m^3
Molar mass of dry air	:	$M_{\text{Air}} = 0.029$ kg/mol
Universal gas constant	:	$R = 8.314$ J/(mol.K)
Sea level pressure	:	$P_0 = 10^5$ Pa
Standard sea level temperature	:	$T_0 = 15$ °C
Heat capacity ratio of air	:	$\gamma = C_p/C_v = 1.4$

Introduction

Tornado is one of the deadliest atmospheric phenomena known to man. It is a violent vortex (rotating column) of air connecting the base of *cumulonimbus*[1] cloud and the ground. A distinct feature of the tornado is its funnel-like *core* or *condensation funnel* (Region II) which is made of small water droplets that condense as they are sucked into the core as shown in Fig. 1(b). This region is defined by the core radius $r_C(z)$ which generally increases with altitude forming the signature funnel-shape of the tornado.

Region I is the region outside tornado core. Region I and II have different velocity distribution profile as we will explore later.

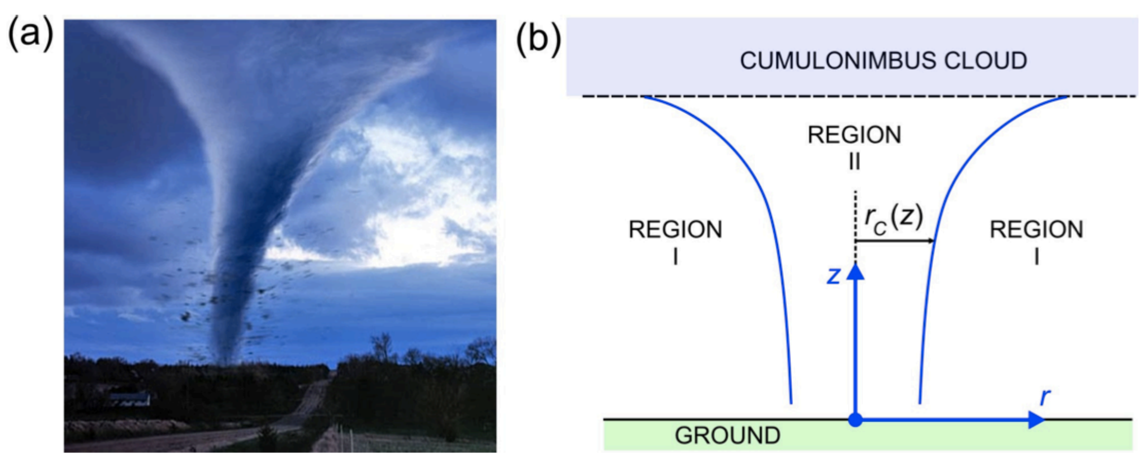

Figure 1: (a) A tornado wreaking havoc in Texas, US. (b) A cross section diagram of a tornado and its coordinate system.

Let us explore the interesting physics of tornado. Using a simple model as shown in Fig. 1(b) and few basic principles, you will try to estimate the rotating speed of tornado, calculate the pressure and temperature inside the tornado and *most interestingly derive the equation for the shape of a tornado $r_C(z)$.*

[1] Cumulonimbus cloud is a towering vertical cloud that is very tall, dense, and involved in thunderstorms and other rainy weather.

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Questions

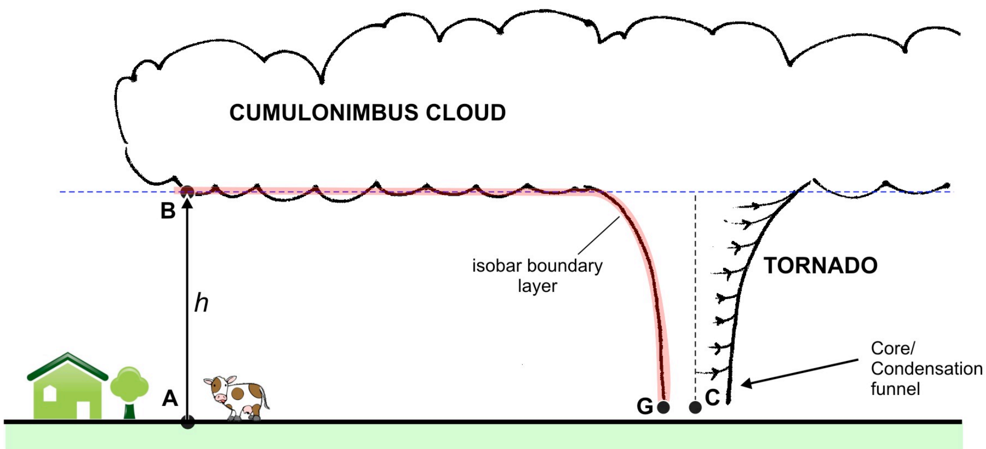

Figure 2: A Tornado landscape

1 The calm weather

We will investigate the atmospheric pressure of the *troposphere* (the lowest part of the atmosphere) where most of the weather phenomena including tornado occurs. Let us start from a calm weather location at point **A** far away from the tornado. At point **A** the pressure is P_0 and temperature T_0 (see Constants and Data).

(a) Assuming ideal gas law, constant gravity acceleration and a constant temperature T_0. Show that the atmospheric pressure as a function of altitude z is:

$$P(z) = P_0 e^{-\alpha z}$$

Express α in terms of the constants listed in "Constants and Data". (0.8 points)

(b) Now we consider a situation where the air density, ρ_{Air}, is constant. Derive the pressure as a function of altitude: $P(z)$! The temperature T drops with altitude z at a linear rate of b. Find b! (1.0 points)

(c) Using your result in (b) calculate the pressure at point **B** on the base of the cumulonimbus! (use $h = 1$ km)

(0.2 points)

2 The shape of tornado

Inside the tornado's core the water vapor condenses into liquid droplets as the air spirals into the core forming *condensation funnel*. The water vapor condenses when the temperature drops below certain point called *dew point*. This temperature drop is caused by pressure drop. Thus the region where the water vapor starts to condense marks a boundary of *equal pressure* called isobar boundary layer that stretches from the base of the cumulonimbus cloud down to the base of tornado (shown as red boundary in Figure 2). This is the boundary between region I and II.

Now only consider region I. Consider a reference point G (Fig. 2) very close to the ground ($z \approx 0$) located at radius r_G. The speed v_G can be treated as the ground rotation speed of the tornado.

We further assume: (i) The tornado is stationary (only has rotation and no translation); (ii) The wind radial velocity is negligible. Velocity v is only tangential (along the circle), not radial. (iii) The wind velocity v is independent of altitude z, it only depends on the radial position r. (iv) We ignore turbulence very close to the ground. (v) We assume air mass density (ρ_{air}) is constant.

(a) Show that in both region I and II along r :

$$\frac{\partial P}{\partial r} = \rho_{\text{air}} \frac{v^2}{r}$$

(0.4 points)

(b) In region I calculate the tangential wind velocity v as a function of r and in terms of v_C and r_C (velocity and radius at the core boundary)at any given altitude (z)! (0.5 points)

(c) Estimate the air speed v_G at the base of tornado at point **G**! (0.5 points)

(d) Derive the shape of the condensation funnel or the tornado core i.e. the function $r_C(z)$, express them in terms of r_G and v_G and altitude z! Plot or sketch this tornado shape in dimensionless quantities z/h vs. r/r_G, where h is the height defined in Fig. 2! (2.0 points)

(e) Most tornadoes look like funnel (the radius is larger at higher altitude) while some is more uniform in diameter like a pipe. Given everything the same, which one do you think has the higher ground rotation speed v_G? (0.5 points)

3 The core of tornado

We will try to calculate the pressure at the center of tornado. Now we will consider both region I and II.

(a) In region II ($r < r_C$) the tornado core behaves as rigid body, derive expression for the (tangential) speed $v(r)$ in this region. Plot the velocity profile from $r = 0$ to ∞!

(1.1 points)

(b) Calculate the pressure at the center of the tornado (point **C**, at the same altitude as point **G**! Use v_G from part 2(c)! (1.2 points)

(c) Estimate the temperature at the center of the tornado (point **C**)! (0.5 points)

(d) Based on your finding in (c) suggest in only few words what could be a possible source of tornado's tremendous energy! (0.3 points)

4 Shall you open or close the windows?

The differential pressure near a tornado is thought to cause poorly ventilated houses to "explode" even though the tornado is only passing at a distance. Therefore some people suggest that the windows have to be opened to vent or let the pressure in the house equilibrates with outside quickly. However, opening the windows will risk more damage due to debris and projectiles getting into the house.

Consider a house with all windows and openings closed with a flat roof of dimension (width x length x thickness) 15 m x 15 m x 0.1 m and mass density $\rho_{\text{Roof}} = 800$ kg/m^3. The tornado is coming fast and passing at a distance $d = 2r_G$ away from the house.

(a) What is the ratio of the lift force on the roof compared to its weight? (0.8 points)

(b) Shall you open or close the windows? (0.2 points)