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Theoretical 1: Magnetic Monopole

1 Introduction

Our everyday experience shows that the magnetic poles always exist in pairs North and South. Breaking a magnet results in the appearance of a new pair of opposite magnetic poles on the two broken ends. The fundamental laws of physics, however, do not contradict the existence of magnetically charged particles called magnetic monopoles. The magnetic monopole is an object possessing just one magnetic pole, either North, or South, which are analogues of the positive and negative electric charges respectively. Thus, the magnetic field of the monopole is similar to the electric field created by a static electric charge, i.e. its force lines begin or end at the point where the monopole is located. This property is in contrast to the closed force lines of the magnetic field created by permanent magnets (magnetic dipoles) and electric currents. The concept of magnetic monopole was introduced in 1932 by the famous physicist Paul Dirac. On the basis of quantum mechanics he proved that the existence of magnetic monopoles can explain the existence of the elementary electric charge. That is why the physicists do not cease their efforts to discover magnetic monopoles experimentally.

In the following questions you are going to establish some properties of the magnetic monopoles by analyzing simple model situations (though experiments). You may assume that all laws of physics known to you apply to the magnetic monopoles, except the statement for closed force lines of the magnetic field. The velocities considered in this problem are much smaller than the speed of light and, therefore, you may neglect the relativistic effects on time, length and mass.

Use the following physical constants in your solution:

magnetic permeability of vacuum:	$\mu_0 = 4\pi \times 10^{-7}$ H/m ;
electric permittivity of vacuum:	$\epsilon_0 = 8.85 \times 10^{-12}$ F/m ;
speed of light:	$c = 2.998 \times 10^{8}$ m/s ;
elementary electric charge:	$e = 1.602 \times 10^{-19}$ C ;
Planks constant:	$h = 6.626 \times 10^{-34}$J.s .

2 Questions

1. When exposed to external magnetic field of induction $\mathbf{B}$, a monopole of magnetic charge q_m experiences a force:

$$\mathbf{F} = q_m \mathbf{B} \tag{1}$$

   (a) Derive the unit of magnetic charge in terms of the basic SI units: kilogram, meter, second, ampere. (0.8 points)

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2. Electric current I circulates along a circular loop of radius a. A monopole of magnetic charge q_m is situated on the axis of the loop at a point of coordinate z relative to its center, as shown in Figure 1 . The positive direction of the axis Z and the direction of current circulation are related through the right-hand rule.

   (a) Find out an expression for the z-component F_z of the force acting on the monopole. (1.1 points)

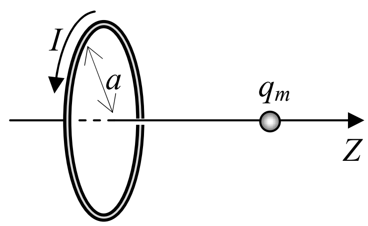

Figure 1: Question 2

3. When at rest, the magnetic monopole creates static magnetic field, similar to the electric field produced by a static electric charge. The magnetic induction $\mathbf{B}$ at a point of position- vector $\mathbf{r}$, relative to the monopole (see Figure 2), is given by the equation:

$$\mathbf{B} = \frac{k_m q_m \mathbf{r}}{r^3}, \tag{2}$$

where k_m is a coefficient of proportionality and $r = |\mathbf{r}|$.

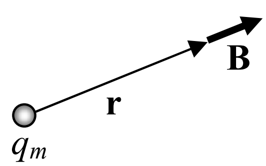

Figure 2: Question 3

   (a) By analyzing the system described in Question 2, express the unknown coefficient k_m through the provided fundamental constants. (1.6 points)

   (b) Formulate by means of equation the Gauss law for the flux Φ of the magnetic induction created by the magnetic monopole. (0.5 points)

4. A moving electric charge creates magnetic field. Likewise, the moving magnetic monopole produces electric field with circular force-lines (i.e. a vortex field) concentric with the direction of motion of the monopole (see Figure 3). Consider a monopole of magnetic charge q_m moving along a straight line with a constant velocity $\mathbf{v}$.

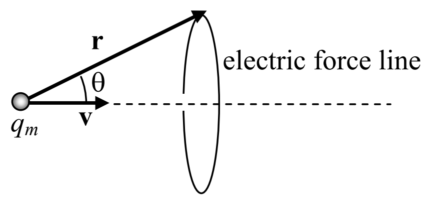

Figure 3: Question 4

(a) Derive an expression for the intensity $\mathbf{E}$ of the electric field created by the monopole in a point of position-vector $\mathbf{r}$ making an angle θ with the vector of velocity, as shown in Figure 3. Use vector notations in your final answer in order to specify both, the magnitude, and the direction of the electric field. (1.7 points)

(b) Suppose that a positive electric charge q_e and a positive magnetic charge q_m are moving toward you, perpendicularly to the sheet of paper. Draw an arbitrary force line in your answer sheet for the magnetic field created by the electric charge. Draw separately another arbitrary force line for the electric field created by the magnetic charge. Indicate the directions of the two lines. (0.4 points)

5. The analogy between electric and magnetic charges is found also in the way they interact with external magnetic and electric fields respectively. Similarly to the Lorentz force acting on an electric charge moving in magnetic field, the magnetic charge experiences a force when it moves in electric field.

   (a) Propose and analyze a thought experiment in order to derive an expression for the Lorentz force acting on a monopole of magnetic charge q_m moving with a velocity $\mathbf{v}$ in electric field of intensity $\mathbf{E}$. Use vector notations in your final answer in order to specify both, the magnitude and the direction of the force. When describing your thought experiment, use proper drawings and short comments to them instead of a lengthy text. (1.0 points)

6. A point particle of electric charge q_e is confined to move along a circle without any resistance or friction, as shown in Figure 4. A monopole of magnetic charge q_m passes through the plane of the circle by moving along its axis Z from $z \to -\infty$ to $z \to \infty$.

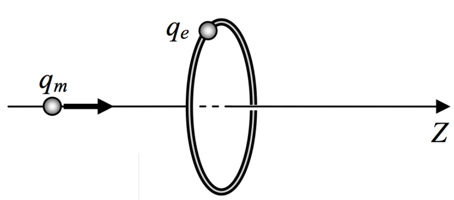

Figure 4: Question 6

(a) Obtain an expression for the change in Zcomponent, ΔL_z, of the angular momentum of the electrically charged particle during the whole motion of the magnetic monopole. Express your answer in terms of q_e, q_m and fundamental constants only. (1.3 points)

7. In his famous work on magnetic monopoles Paul Dirac has argued that if just one magnetic monopole existed in the Universe, all electric charges should be multiple of a specific elementary electric charge, whose magnitude is related to the magnetic charge of that monopole. Historically, it is the first hypothesis in physics, which explains the existence of the elementary electric charge.

   Consider the system described in Question 6, assuming that all magnetic monopoles existing in the Nature have magnetic charges of the same magnitude, $+q_m$ and $-q_m$ respectively.

   (a) By applying the concepts of quantum physics to the motion of electrically charged particle along the circular orbit, derive a relationship between the elementary electric charge e, assumed to be the charge of the electron, and the magnetic charge q_m of the monopole. Calculate q_m numerically. (1.1 points)

   (b) The electron possesses a self magnetic moment of $p_m = 9.274 \times 10^{-24}$ A.m^2 . By assuming that the magnetic properties of the electron are due to a pair of spatially separated point magnetic monopoles of opposite magnetic charges, $+q_m$ and $-q_m$ respectively; calculate the distance d between these monopoles. (0.5 points)

Useful math :

- The solid angle Ω enclosed by a cone of half-opening angle θ (see the Figure 5) is: $\Omega = 2\pi(1 - \cos(\theta))$

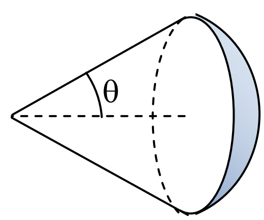

Figure 5: Solid Angle

- Depending on your approach to the solution you may need the following integral:

$$\int_{-\infty}^{\infty} \frac{dz}{(z^2 + a^2)^{3/2}} = \frac{2}{a^2}. \tag{3}$$